Panel Data I

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Applied Quantitative Methods II

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- Compare fixed effects and random effects; know when to use which
- Cluster standard errors correctly in panel settings

Roadmap

What is Panel Data?

The Problem: Unobserved Heterogeneity

Fixed Effects

Two-Way Fixed Effects

Random Effects and the FE/RE Choice

Clustered Standard Errors

Wrap-up

Panel data: the basic structure

Unit i	**Time** t	y_{it}	x_{it}
1	2010	0.42	12.1
1	2011	0.51	13.0
1	2012	0.48	12.7
2	2010	0.61	9.4
2	2011	0.59	9.8
2	2012	0.64	10.2

⋮

- N units, each observed at T time points
- Data indexed (i, t): unit i at time t

Panel data: examples

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- **Firms**: quarterly earnings reports for publicly traded companies
- **Running example**: US state-level presidential approval $\times$ years

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  - → Units may differ in ways we cannot measure
  - → Panel structure lets us "absorb" those differences

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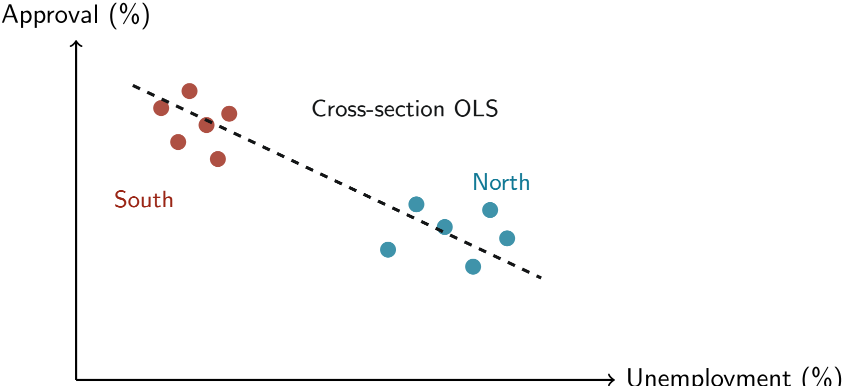
Motivating example: presidential approval

Does unemployment drive down presidential approval?



The cross-sectional slope is negative.

Does that mean unemployment *causes* lower approval?

What else might explain this pattern?

The problem: unit-level confounders

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$$y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$$

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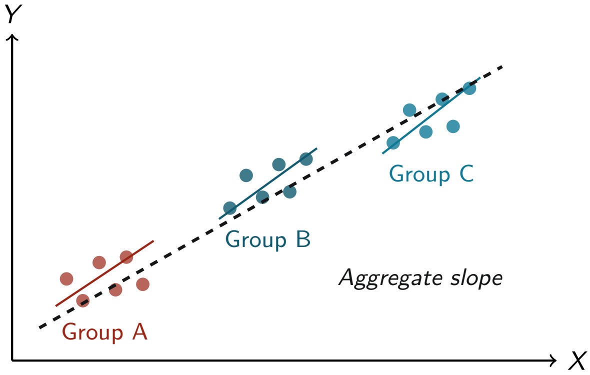
- Cross-section OLS ignores $\alpha_i \Rightarrow$ omitted variable bias

Simpson's paradox: the intuition



- Within each group: positive slope
- Cross-section OLS: also positive, but for the **wrong reason**
- The group-level differences dominate the estimate

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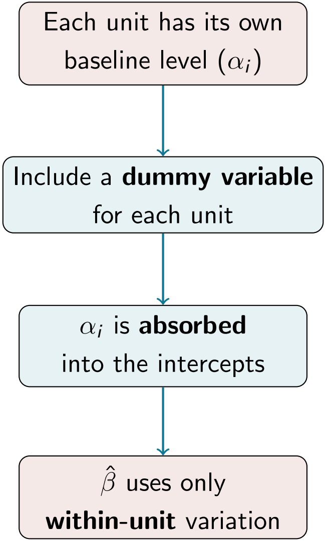
Fixed effects: the key idea



The within (demeaning) estimator

Starting from $y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$, subtract unit means:

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- α_i **cancels out** — the unit effect is gone
- Regressing $\tilde{y}_{it}$ on $\tilde{x}_{it}$ gives the FE estimator
- Uses only variation *within* each unit over time
- Works because α_i is constant: $\bar{\alpha}_i = \alpha_i$

FE = dummies for each unit

- Mathematically equivalent to including unit dummies:

$$y_{it} = \sum_{i=1}^{N} \alpha_i D_i + \beta x_{it} + \varepsilon_{it}$$

where $D_i = 1$ if observation belongs to unit i

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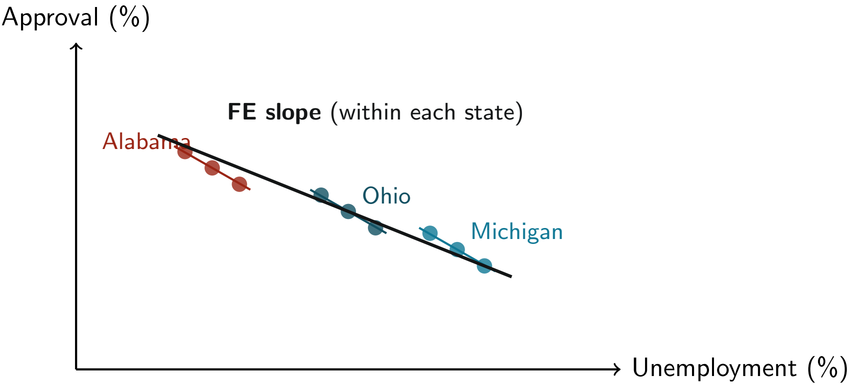
- **Key implication**: cannot estimate effect of **time-invariant** variables
  - → If z_i does not vary over time, it is collinear with D_i
  - → Example: "South" dummy, gender, country of birth

Presidential approval: what FE does



- Each state has its own intercept; the slope is shared
- FE estimates: as *this state's* unemployment rises, its approval falls

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- Tables with `modelsummary()` work seamlessly with `feols` objects

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FE controls for everything time-invariant.

But what if something happens in 2008 that affects *all* states simultaneously?

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- Solution: add **time fixed effects** γ_t

Two-way fixed effects model

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Type of variation	**Unit FE**	**Two-way FE**
Time-invariant unit differences	absorbed	absorbed
Unit-invariant time shocks	not absorbed	absorbed
Within-unit, across-time variation	used for $\hat{\beta}$	used for $\hat{\beta}$

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- But more credible: fewer threats to identification

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- $\hat{\beta}_{\text{Apct}}$: compares same instructor's courses, not different instructors

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Random effects: a different assumption

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- If this fails: RE is **biased**; FE is consistent

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- If this fails: RE is **biased**; FE is consistent
- Unlike FE: can estimate effects of **time-invariant** variables

FE vs. RE: the tradeoff

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Consistency	Always (if $T \to \infty$)	Only if $\eta_i \perp X$
Efficiency	Less efficient	More efficient
Time-invariant vars	Cannot estimate	Can estimate

- If you are unsure: **use FE**

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- More conservative, more credible
- RE requires an untestable assumption; FE does not

Hausman test: FE vs. RE

- **Hausman test** (1978): formally test whether $\eta_i \perp x_{it}$

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- H_1: correlation exists (only FE is consistent)

Hausman test: FE vs. RE

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Roadmap

What is Panel Data?

The Problem: Unobserved Heterogeneity

Fixed Effects

Two-Way Fixed Effects

Random Effects and the FE/RE Choice

Clustered Standard Errors

Wrap-up

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- Solution: **cluster standard errors by unit**

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  - → If state gets the treatment, cluster by state
  - → If individual gets the treatment, cluster by individual

Comparing specifications: a template

	(1) OLS	(2) Unit FE	(3) TWFE
Unemployment	-0.82^{***}	-0.51^{**}	-0.43^{**}
	(0.15)	(0.18)	(0.16)
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Year FE	No	No	Yes
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- Preferred specification: (3)

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- **Two-way FE**: add time dummies to remove common shocks
- **FE vs. RE**: use FE when unit effects may correlate with X; use Hausman test
- Always **cluster standard errors** by unit

For next session

- Complete Assignment 5
- Read the assigned paper using panel FE
- Next session: Panel Data II
  - → Difference-in-Differences (DiD)
  - → Event studies
  - → Staggered treatment timing
  - → Recent advances in DiD (Callaway–Sant'Anna, etc.)

Questions?